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# Method for estimating capacity and measures of effectiveness of two-lane highway work zones 

# Método para determinação da capacidade e de medidas de desempenho em zonas de obras de rodovias de pista simples 

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#### Abstract

The present study proposes a method for determining the capacity and measures of effectiveness of two-lane highway work zones. Therefore, a deterministic mathematical formulation was developed based on Queue Theory and on similarities with the queue discharge flow that occurs with the use of traffic lights, which allows for the estimation of measures of effectiveness such as delay time and platoon size. Equations were also developed to determine the capacity and the maximum length of the work zones based on acceptable values of the chosen measures of effectiveness. To validate the method, a set of traffic data collected along segments of a highway during the execution of construction works was used to calibrate and validate the traffic simulator Aimsun Next. With the simulator calibrated, a set of synthetic data was generated based on the simulation of different hypothetical scenarios, which served to obtain variables of the problem and to validate the proposed method. Finally, measures of effectiveness such as cycle time, platoon size, and delay time - obtained from the proposed mathematical model and the field - were compared, revealing that the model produced very satisfactory results.


## RESUMO

O presente estudo propõe um método para determinação da capacidade e de medidas de desempenho em zonas de obras de rodovias de pista simples. Para tanto, desenvolveu-se uma formulação matemática determinística, baseada em Teoria de Filas e nas semelhanças com a dissipação de veículos que ocorre em semáforos, o que permite estimar algumas medidas de desempenho como atraso e tamanho de pelotão. Foram desenvolvidas também equações para determinar a capacidade e o comprimento máximo de zona de obras, a partir de valores aceitáveis de medidas de desempenho. Para validação do método, utilizou-se um conjunto de dados de tráfego coletados em trechos de uma rodovia durante a execução de obras para calibrar e validar o simulador de tráfego Aimsun Next. Com o simulador calibrado, foi gerado um conjunto de dados sintéticos, a partir da simulação de diversos cenários hipotéticos, que serviram tanto para obter algumas variáveis do problema quanto para validar o método proposto. Por fim, foram comparadas medidas de desempenho - obtidas do modelo matemático proposto e em campo - tais como ciclo, tamanho de pelotão e atraso, as quais apresentaram resultados bastante aderentes.

## 1. INTRODUCTION

The execution of construction works in segments of two-lane highways requires the closure of the road in one direction, while the remaining lane is used alternately for both directions of traffic. In this operation, called stop-and-go, traffic control devices are installed at each end,
blocking the traffic flow in one direction until vehicles from the opposite direction complete the travel.

CONTRAN (2017) recommends determining traffic flow interruption times by considering the comfort and safety of the users to minimize retention queues and avoid waiting times longer than 30 minutes during traffic flow alternation. The determination of measures related to the vehicle queue and waiting time in a construction site, however, is quite complex and depends on several factors, such as the traffic demand on the segment, the extent of the intervention, and the geometric characteristics of the segment.

Although stop-and-go operations are quite frequent on two-lane highways, research on this subject is still incipient (Zhu et al., 2017). The Highway Capacity Manual (HCM) incorporated for the first time, in its sixth edition (TRB, 2016), operational analyses of sections along construction works, based on the results of the NCHRP 03-107 project, within the scope of the National Cooperative Highway Research Program (Schoen et al., 2015). However, the methods incorporated in HCM-6 were restricted to multilane highways, where the execution of construction works does not require stop-and-go operations.

Among the studies related to two-lane segments, different approaches were found for evaluating traffic performance in interventions of this type, such as deterministic (Cassidy and Han, 1993) or stochastic (Son, 1999) and microscopic simulations (Shibuya et al., 1996; Washburn et al., 2008; Zhu et al., 2017; Hua et al., 2019). However, the literature still lacks a more comprehensive exploratory analysis that incorporates the specificities inherent to the problem and thus a suitable method to predict the impacts of the intervention under various conditions. Most of the studies found in the literature are limited to the evaluation of case studies, without a more generalist approach to the mathematical relationships involved in the process.

Some studies have applied traffic simulation techniques (Shibuya et al., 1996; Washburn et al., 2008) to incorporate regression models to determine the delay time as a function of a series of input parameters, such as traffic flow, percentage of heavy vehicles, and work zone length. The studies by Zhu et al. (2017) and Hua et al. (2019) compared the performances of stop-andgo operations adopting fixed and dynamic green intervals. Both studies concluded that the best strategy is to maintain the traffic flow released in each direction only during the queue dissipation state.

In the Brazilian context, the São Paulo State Transportation Agency (ARTESP) developed a Technical Specification that defines the guidelines for the analysis of the expected traffic behavior and presents a method for determining the level of service in work zones (ARTESP, 2019). The formulation described considers as input data such as traffic (directional traffic flows and percentage of heavy vehicles), geometric configuration (lane width, lateral clearance, and posted speed limit), and work zone information (work zone length, road marking) to estimate the segment capacity and calculate measures of effectiveness including the maximum queue length, average delay, and level of service.

Mathematical modeling considers the similarities between the stop-and-go operation and traffic light operation to measure the stages of release and blocking of the highway, establishing green intervals per direction and cycle. However, some peculiarities of the stop-and-go operation are not properly addressed in the ARTESP (2019) method, such as the influence of traffic flow and clearance time on the dimensioning of traffic light times, with work zone lengths up to 500 m . Due to these divergences, the application of the ARTESP method generates
inconsistent results, such as cycle times shorter than the sum of the green intervals calculated for each traffic direction, and, consequently, estimates a highway capacity that may be higher than the saturation flow established for the segment.

Thus, the objective of this study is to propose a method for determining the capacity and measures of effectiveness of two-lane highway work zones. The mathematical formulation of the proposed method was developed based on queuing theory, considering the similarities between a stop-and-go operation and a traffic light operation.

## 2. PROPOSED METHOD

This study adopted mathematical modeling and traffic microsimulation to establish parameters and variables that reproduce the traffic conditions of two-lane highway segments during the execution of construction works. Figure 1 summarizes the methodological steps performed in this study.


Figure 1. Method steps for estimating capacity and measures of effectiveness of two-lane highway work zones

Initially, a mathematical modeling of the problem was proposed, and variables that influence the measures of effectiveness of stop-and-go interventions for two-lane highways were identified through a literature review. Subsequently, traffic data were collected in two-lane highway segments during the execution of the works, measuring variables of the traffic stream and the queue discharge, such as the discharge traffic flow, heavy vehicle equivalence factor,
average travel speed, and lost time, which are crucial for the application of the developed mathematical model and the calibration and validation of the traffic microsimulation model, Aimsun Next. Once the simulator was calibrated, several replications of predefined test scenarios were performed, and the results were used to calculate the variables involved and validate the measures of effectiveness obtained through the application of the developed mathematical model.

Finally, the measures of effectiveness obtained from the proposed mathematical method and the microsimulation of the test scenarios were compared during the validation process. If it was indicated that the mathematical models were not satisfactory, the modeling of the variables that justify the difference between the measures of effectiveness was resumed.

## 3. MATHEMATICAL FORMULATION

The blocking of a lane in segments of two-lane highways converts the uninterrupted flow condition into an interrupted traffic condition. Figure 2 illustrates the configuration of a construction deviation in a two-lane highway segment. In this example, the construction site is in the traffic lane corresponding to Direction 1, and the vehicles traveling in that direction must travel along the opposite lane. In this sense, there must be access control in the impeded segment to avoid vehicle collision during the traffic operation.


Figure 2. Example of a work zone on a two-lane highway segment (ARTESP, 2019)

Using traffic control devices (flags or signs) located at each end, the operation of closing the road is similar to the operation of traffic lights. Both serve to control the access of conflicting traffic flows to a specific location, be it an intersection, as is the case of traffic lights, or a work zone, which requires a stop-and-go intervention. In general, there are four phases involved in this system:

- $\boldsymbol{G}_{1}$ : green interval of Direction 1, i.e., the period in which the traffic is released at the control point of Direction 1 and blocked in Direction 2.
- $\boldsymbol{C} \boldsymbol{T}_{1}$ : clearance time of the vehicles in Direction 1, i.e., the interval necessary for the vehicles to travel in the segment. In this interval, both control points are blocked.
- $\boldsymbol{G}_{2}$ : green interval of Direction 2, that is, the period in which the traffic is released at the control point of Direction 2 and blocked in Direction 1.
- $\boldsymbol{C} \boldsymbol{T}_{2}$ : clearance time of vehicles in Direction 2; both control points are blocked.

In the following sections, the formulations for the dimensioning of the phases and the calculation of measures of effectiveness such as platoon size and average delay are presented. Equations are also described for determining the maximum work zone length and capacity based on platoon size and delay limits.

### 3.1 Clearance time

The dimensioning of the phases $C T_{1}$ and $C T_{2}$ depends exclusively on the work zone length and the average travel speed of the vehicles in the segment:

$$
\begin{align*}
& C T_{1}=\frac{L}{S_{1} / 3.6}  \tag{1}\\
& C T_{2}=\frac{L}{S_{2} / 3.6} \tag{2}
\end{align*}
$$

where $\quad C T_{1}, C T_{2}$ : clearance times in Directions 1 and 2 , in s ;
$L: \quad$ work zone length, in $m$
$s_{1}, s_{2}: \quad$ average travel speeds in Directions 1 and 2 , in $\mathrm{km} / \mathrm{h}$.

### 3.2. Cycle and green intervals

The determination of the green intervals $G_{1}$ and $G_{2}$, in turn, depends on a series of factors, such as traffic volume per direction, saturation flow, and cycle time. The difference between the operations of a stop-and-go system and a fixed-time traffic light must be understood.

With fixed-time traffic lights, the dimensioning of the green interval should provide an additional interval, in addition to the average saturation levels, to support the randomness of the demand for vehicles per cycle, which naturally decreases the efficiency of the traffic light. In practice, this additional time is responsible for reducing the degree of saturation of the intersection and, consequently, mitigating the effects of the delay incremental component responsible for estimating the probability of the queue not being fully served during the green interval.

The stop-and-go intervention, in turn, can adapt the operations cycle according to the demand in each direction, dynamically varying the green intervals according to the corresponding demand. Its basic principle is to release traffic until all vehicles queued in each direction are served, which represents the optimal operation procedure of these interventions. Figure 3 illustrates the two situations represented in queue diagrams, which guide all the following steps of the proposed formulation. Note that the additional green interval of the fixedtime traffic light thus promotes a decrease in the traffic light efficiency, while the stop-and-go operation works dynamically, adapting the green interval until all vehicles in the queue are served.

For simplification, the calculations of the proposed mathematical formulation were obtained from deterministic values, observing the optimal green interval conditions. As previously mentioned, one of the characteristics of the stop-and-go operation is its ability to adapt to the variation in demand, with no loss of efficiency during off-peak times. This means that the sum of the green intervals in all cycles in a period, for each direction, is proportional to the average traffic flow along the segment, even though it varies in a compensatory manner around the average. For this reason, it is understood that the determination of measures of effectiveness based on average values is representative and that the formulation of the additional delay component for the surplus queue of the cycle does not apply.


Figure 3. Queue diagram for fixed-time traffic lights and stop-and-go operation

The fundamental premise of the proposed method consists of dimensioning the green interval until the cumulative arrival and departure curves are equal, at which point the reversal of the direction of traffic flow in the impeded segment begins. It is important to note that, in this case, the queues are not fully served in each cycle. In mathematical terms, the capacity offered must be equal to the demand for vehicles in each direction. Thus, the following equation is used to determine the effective green interval for each direction:

$$
\begin{equation*}
\frac{g_{i}}{c}=\frac{v_{i}}{Q_{i}} \Rightarrow g_{i}=\frac{v_{i} \times C}{Q_{i}} \tag{3}
\end{equation*}
$$

where $\quad C$ : average cycle period, in s ;
$g_{i}$ : effective green interval for Direction $i$, in $s$;
$v_{i}$ : traffic flow of Direction $i$, in passenger cars per hour ( $\mathrm{pc} / \mathrm{h}$ );
$Q_{i}$ : saturation flow for Direction $i$, in pc/h.
The missing component for determining the cycle time is called lost time ( $L T$ ), which includes the clearance times for each direction ( $C T_{1}$ and $C T_{2}$ ) and the lost time for the traffic release and the start of the first vehicles $\left(l_{s}\right)$, which includes the interval between the passage of the last vehicle at the end of the segment under construction and the start of the first vehicle in the queue and the excess headway time of the first vehicle in relation to the saturation headway:

$$
\begin{equation*}
L T=C T_{1}+C T_{2}+2 l_{s}=\frac{L}{s_{1 / 3.6}}+\frac{L}{S_{2} / 3.6}+2 l_{s} \tag{4}
\end{equation*}
$$

where $\quad L T$ : total lost time, in s ;
$C T_{1}, C T_{2}$ : clearance times in Directions 1 and 2, in s;
$l_{s}: \quad$ lost time for traffic release and vehicle startup, in $s$.
$L: \quad$ work zone length, in $m$.
The following equations show the deduction of the cycle calculation, which depends on the degree of total saturation of the segment $(v / Q)$ and the work zone length, a parameter that is also used to determine lost time.

$$
\begin{equation*}
C=g_{1}+g_{2}+L T=\frac{v_{1} \times C}{Q_{1}}+\frac{v_{2} \times C}{Q_{2}}+L T=\frac{L T}{1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)} \tag{5}
\end{equation*}
$$

### 3.3. Average platoon size

From the determination of the cycle time $(C)$ and, consequently, of the effective green intervals, it is possible to determine the average number of vehicles served per cycle in each direction,
which represents one of the main negative effects of the stop-and-go operation. By stopping the vehicles at the ends of the segment under construction and then releasing them, platoons are formed that impair the traffic conditions of the users. The greater the accumulation of vehicles in this process, the greater the effects of heavy vehicles on downstream traffic, requiring more overtaking and more time to disperse the platoons.

From the dimensioning of the operational phases of the stop-and-go intervention, it is possible to calculate the average platoon size, which consists of dividing the directional traffic flow - which is the total number of vehicles that experienced a queue in one hour - by the number of cycles in one hour, as shown in the following equation:

$$
\begin{equation*}
P_{i}=\frac{v_{i}}{\frac{3,600}{c}}=v_{i} \times \frac{c}{3,600} \tag{6}
\end{equation*}
$$

where $\quad P_{i}: \quad$ average platoon size for Direction $i$, in pc.

### 3.4. Average delay

The calculation of the average delay time is also based on the queue diagram (F Figure 4). In this image, the following variables are represented:
$P_{i}$ : platoon size or number of vehicles served per cycle in Direction $i$;
$g_{i}$ : effective green interval for Direction $i$, in s;
$C: \quad$ cycle time, in s.


Figure 4. Stop-and-go operation for delay estimation

Graphically, the area of the gray triangle formed between the arrival and departure curves corresponds to the total delay of the vehicles in the cycle. Therefore, the following equation is formulated:

$$
\begin{equation*}
d T_{i}=\frac{P_{i} \times\left(C-g_{i}\right)}{2} \tag{7}
\end{equation*}
$$

where $\quad d T$ : total delay time of Direction $i$, in $\mathrm{pc} / \mathrm{s}$;
This means that the average delay of vehicles served in Direction $i\left(d_{i}\right)$ is equivalent to half the effective red interval of this direction, as shown in the equation below:

$$
\begin{equation*}
d_{i}=\frac{d T_{i}}{P_{i}}=\frac{\left(C-g_{i}\right)}{2} \tag{8}
\end{equation*}
$$

In turn, the average delay $d$, in seconds, can be obtained by the weighted average of the directional delays by the corresponding traffic flows:

$$
\begin{equation*}
d=\frac{d_{1} \times v_{1}+d_{2} \times v_{2}}{v_{1}+v_{2}} \tag{9}
\end{equation*}
$$

### 3.5. Capacity

The capacity, specifically in the case of the stop-and-go operation, consists of the sum of the maximum directional traffic flow from pre-established conditions, such as speed, saturation flow, and work zone length. It is also conditioned on the definition of acceptable operational parameters, such as limit values of average platoon size ( $P_{\text {lim }}$ ) and average delay ( $d_{\text {lim }}$ ). The following sections present the deduction of the formulas for determining capacity as a function of these conditions separately.

### 3.5.1. Capacity versus platoon size

The average platoon size in each direction depends directly on the corresponding directional traffic flow and cycle time, as shown in the following equation:

$$
\begin{equation*}
P_{i}=\frac{v_{i} \times C}{3,600} \tag{10}
\end{equation*}
$$

Considering that the largest platoon is formed in the direction of the main traffic flow, denoted as $v_{1}$, the secondary traffic flow can be determined as follows:

$$
\begin{equation*}
v_{2}=k v_{1}, k \leq 1 \tag{11}
\end{equation*}
$$

Thus, the cycle can be calculated as a function of the total lost time $L T$, the saturation flows $Q_{1}$ and $Q_{2}$, and the main traffic flow $v_{1}$ :

$$
\begin{equation*}
C=\frac{L T}{1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)} \Rightarrow C=\frac{L T}{1-v_{1}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)} \tag{12}
\end{equation*}
$$

This implies that equation (10) takes the following form:

$$
\begin{equation*}
P_{1}=\frac{v_{1}}{3,600} \times \frac{L T}{1-v_{1}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)} \Rightarrow \frac{v_{1} L T}{3,600}=P_{1}\left(1-v_{1}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)\right) \tag{13}
\end{equation*}
$$

The main traffic flow can then be written as a function of the average platoon size:

$$
\begin{equation*}
\frac{v_{1} L T}{3,600}=P_{1}-P_{1} v_{1}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right) \Rightarrow v_{1}=\frac{P_{1}}{\frac{L T}{3,600}+P_{1}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)} \tag{14}
\end{equation*}
$$

Let the defined capacity (cap) be the sum of the maximum directional traffic flows ( $v 1$,max and $v_{2, \text { max }}$ :

$$
\begin{equation*}
c a p=v_{1, \max }+v_{2, \max }=(k+1) v_{1, \max } \tag{15}
\end{equation*}
$$

Applying this expression to equation (14) and considering the platoon size limit $P_{\lim }$ as a condition for determining the capacity, the following equation can be used to estimate the capacity:

$$
\begin{equation*}
\text { cap }=(k+1) v_{1, \max }=\frac{(k+1) P_{\text {lim }}}{\frac{L T}{3,600}+P_{\text {lim }}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)} \tag{16}
\end{equation*}
$$

where cap: work zone capacity, considering the two traffic directions, in $\mathrm{pc} / \mathrm{h}$;
$P_{\text {lim }}$ : platoon size limit, in pc;
$Q_{1}, Q_{2}$ : saturation flows for Directions 1 and 2, in $\mathrm{pc} / \mathrm{h}$;
LT: total lost time, in s;
$k: \quad$ proportion of the traffic flow in the secondary direction in relation to the main flow.
Notably, the determination of the capacity depends on a series of variables related to the problem, such as directional saturation flows ( $Q_{1}$ and $Q_{2}$ ) and lost time ( $L T$ ), which, in turn, depend on the work zone length and the average travel speed in each direction, in addition to the limitation of the platoon size limit ( $P_{\text {lim }}$ ).

### 3.5.2. Capacity versus average delay

The average delay of users who pass the work zone can be obtained as a weighted average of the delays of the corresponding directional traffic flows:

$$
\begin{equation*}
d=\frac{d_{1} v_{1}+d_{2} v_{2}}{v_{1}+v_{2}} \tag{17}
\end{equation*}
$$

Again, for capacity calculation, the secondary traffic flow $v_{2}$ is defined as a proportion $k$ of the main traffic flow, $v_{1}$ :

$$
\begin{equation*}
v_{2}=k v_{1}, k \leq 1 \tag{18}
\end{equation*}
$$

From this formulation, the equation (17) can be described as a function of the average directional delay and the $k$ ratio, as shown in the following equation:

$$
\begin{equation*}
d=\frac{d_{1} v_{1}+k d_{2} v_{1}}{v_{1}+k v_{1}}=\frac{d_{1} v_{1}+k d_{2} v_{1}}{v_{1}+k v_{1}}=\frac{d_{1}+k d_{2}}{k+1} \tag{19}
\end{equation*}
$$

Recall that the average directional delay depends on the cycle and green intervals:

$$
\begin{equation*}
d_{i}=\frac{c-g_{i}}{2} \tag{20}
\end{equation*}
$$

The green interval can be defined as follows:

$$
\begin{equation*}
\frac{g_{i}}{c}=\frac{v_{i}}{Q_{i}} \Rightarrow g_{i}=\frac{C v_{i}}{Q_{i}} \tag{21}
\end{equation*}
$$

Thus, equation (20) can be rewritten as follows:

$$
\begin{equation*}
d_{i}=\frac{c-\frac{C v_{i}}{Q_{i}}}{2}=\frac{c}{2}\left(1-\frac{v_{i}}{Q_{i}}\right) \tag{22}
\end{equation*}
$$

Rewriting equation (19) to include equations (12) and (22) results in the following expression:

$$
\begin{equation*}
d=\frac{c\left[\left(1-\frac{v_{1}}{Q_{1}}\right)+k\left(1-\frac{k v_{1}}{Q_{2}}\right)\right]}{2(k+1)} \Rightarrow 2 d(k+1)=\frac{L T\left[k+1-v_{1}\left(\frac{1}{Q_{1}}+\frac{k^{2}}{Q_{2}}\right)\right]}{1-v_{1}\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)} \tag{23}
\end{equation*}
$$

Isolating the main directional traffic flow $v_{1}$, the following equation is obtained:

$$
\begin{equation*}
v_{1}=\frac{1-\frac{L T}{2 d}}{\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)-\frac{v_{1} L T}{2 d(k+1)}\left(\frac{1}{Q_{1}}+\frac{k^{2}}{Q_{2}}\right)} \tag{24}
\end{equation*}
$$

Adopting the formulation of the capacity defined in equation (15) and considering the average delay limit $d_{\text {lim }}$ as a condition for determining capacity, the following equation is obtained to estimate capacity:

$$
\begin{equation*}
a p=(k+1) v_{1, \max }=\frac{(k+1)\left(1-\frac{L T}{2 d_{l i m}}\right)}{\left(\frac{1}{Q_{1}}+\frac{k}{Q_{2}}\right)-\frac{L T}{2 d_{l i m}(k+1)}\left(\frac{1}{Q_{1}}+\frac{k^{2}}{Q_{2}}\right)} \tag{25}
\end{equation*}
$$

where cap: work zone capacity, in $\mathrm{pc} / \mathrm{h}$, considering the two traffic directions;
$d_{\text {lim }}$ : delay average limit, in s ;
$Q_{1}, Q_{2}$ : saturation flows for Directions 1 and 2 , in $\mathrm{pc} / \mathrm{h}$;
LT: $\quad$ total lost time, in s;
$k: \quad$ proportion of the traffic flow of the secondary direction in relation to the main traffic flow.

### 3.6. Maximum work zone length

The formulation for dimensioning the maximum length of work zones is analogous to the formulation for capacity estimation and is also based on constraints such as average platoon size limit ( $P_{\text {lim }}$ ) and average vehicle delay time ( $d_{\text {lim }}$ ). The formulations for determining the maximum work zone length as a function of each of these variables are deduced in the following sections.

### 3.6.1. Maximum work zone length versus platoon size

The average platoon size $P_{\mathrm{i}}$ in direction $i$ is defined by the relation between the directional traffic flow $v_{\mathrm{i}}$ (in $\mathrm{pc} / \mathrm{h}$ ) and the number of cycles during a 1-h interval. Considering that the largest platoon is formed in the direction of the main traffic flow, denoted as $v_{1}$, the equation to determine the cycle time (in s) is as follows:

$$
\begin{equation*}
P_{1}=\frac{v_{1} \times C}{3,600}=\frac{v_{1}}{3,600} \times \frac{L T}{1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)} \tag{26}
\end{equation*}
$$

Isolating the total lost time ( $L T$ ), the following equation is obtained:

$$
\begin{equation*}
L T=\left(\frac{3,600 P_{1}}{v_{1}}\right)\left[1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)\right] \tag{27}
\end{equation*}
$$

Applying equation (4), which measures the total lost time in a stop-and-go cycle as a function of the work zone length $L$, the directional velocities $s_{1}$ and $s_{2}$, and the lost time to release the traffic ( $l_{s}$ ), the equation takes the following form:

$$
\begin{equation*}
3.6 L\left(\frac{1}{s_{1}}+\frac{1}{s_{2}}\right)+2 l_{s}=\left(\frac{3,600 P_{1}}{v_{1}}\right)\left[1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)\right] \tag{28}
\end{equation*}
$$

The dimensioning of the maximum length of the work zone $L_{\text {max }}$, considering the platoon size limit $P$ lim, can be obtained through the following equation:

$$
\begin{equation*}
L_{\text {max }}=\frac{\left(\frac{1000 P_{l i m}}{v_{1}}\right)\left[1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)\right]-\frac{l_{s}}{1.8}}{\left(\frac{1}{s_{1}}+\frac{1}{s_{2}}\right)}=\frac{1000 P_{\text {lim }}\left[1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)\right]-\frac{l_{s} v_{1}}{1.8}}{v_{1}\left(\frac{1}{s_{1}}+\frac{1}{s_{2}}\right)} \tag{29}
\end{equation*}
$$

where $\quad L_{\text {max }}: \quad$ maximum length of the work zone, in m;

$$
\begin{array}{ll}
P_{\text {lim }}: & \text { Platoon size limit, in pc; } \\
Q_{1}, Q_{2}: & \text { saturation flows for Directions } 1 \text { and } 2, \text { in } \mathrm{pc} / \mathrm{h} ; \\
s_{1}, s_{2}: & \begin{array}{l}
\text { average travel speed, in } \mathrm{km} / \mathrm{h}, \text { in Directions } 1 \text { and } 2 \text { in the work zone } \\
\text { segment; }
\end{array} \\
v_{1}, v_{2}: & \begin{array}{l}
\text { traffic flows in Directions } 1 \text { and } 2, \text { in } \mathrm{pc} / \mathrm{h} ; \\
l_{s}:
\end{array} \\
\text { lost time for traffic release and vehicle startup, in s. }
\end{array}
$$

It is important to note that the maximum length of a work zone depends on the average platoon size limit and several other factors, such as the degree of saturation of the segment, lost time for traffic release, traffic flow, and directional average travel speed.

### 3.6.2. Maximum work zone length versus average delay

The average delay of the intervention is the result of the weighting of the directional delays calculated by the corresponding traffic volumes. Substituting the average delay $d_{i}$ into equation (22), the equation can be rewritten as follows:

$$
\begin{equation*}
d=\frac{d_{1} v_{1}+d_{2} v_{2}}{v_{1}+v_{2}}=\frac{\frac{C}{2}\left[\left(1-\frac{v_{1}}{Q_{1}}\right) v_{1}+\left(1-\frac{v_{2}}{Q_{2}}\right) v_{2}\right]}{v_{1}+v_{2}} \tag{30}
\end{equation*}
$$

A single cycle $C$ is described as follows:

$$
\begin{equation*}
C=\frac{2 d\left(v_{1}+v_{2}\right)}{\left[\left(1-\frac{v_{1}}{Q_{1}}\right) v_{1}+\left(1-\frac{v_{2}}{Q_{2}}\right) v_{2}\right]} \tag{31}
\end{equation*}
$$

From equation (4), the cycle time can also be calculated as follows:

$$
\begin{equation*}
C=\frac{3.6 L\left(\frac{1}{s_{1}}+\frac{1}{s_{2}}\right)+2 l_{s}}{1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)} \tag{32}
\end{equation*}
$$

Substituting equation (32) into equation (31) results in the following expression:

$$
\begin{equation*}
\frac{3.6 L\left(\frac{1}{s_{1}}+\frac{1}{s_{2}}\right)+2 l_{s}}{1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{Q_{2}}\right)}=\frac{2 d\left(v_{1}+v_{2}\right)}{\left[\left(1-\frac{v_{1}}{Q_{1}}\right) v_{1}+\left(1-\frac{v_{2}}{Q_{2}}\right) v_{2}\right]} \tag{33}
\end{equation*}
$$

The maximum length of the work zone $L_{\text {max }}$, considering the average delay time limit $d_{\text {lim }}$, can be calculated as follows:

$$
\begin{equation*}
L_{\text {max }}=\left\{\frac{2 d_{l i m}\left(v_{1}+v_{2}\right)\left[1-\left(\frac{v_{1}}{Q_{1}}+\frac{v_{2}}{2_{2}}\right)\right]}{\left[\left(1-\frac{v_{1}}{Q_{1}}\right) v_{1}+\left(1-\frac{v_{2}}{Q_{2}}\right) v_{2}\right]}-2 l_{s}\right\} \times \frac{1}{3.6\left(\frac{1}{s_{1}}+\frac{1}{s_{2}}\right)} \tag{34}
\end{equation*}
$$

| where | $L_{\text {max }}:$ | maximum length of the work zone, in $\mathrm{m} ;$ |
| :--- | :--- | :--- |
| $d_{\text {lim }}:$ | delay average limit, in $\mathrm{s} ;$ |  |
| $Q_{1}, Q_{2}:$ | saturation flows for Directions 1 and 2, in $\mathrm{pc} / \mathrm{h} ;$ |  |
| $s_{1}, s_{2}:$ | average travel speeds in Directions 1 and 2 in the work zone segment, in |  |
|  | $\mathrm{km} / \mathrm{h} ;$ |  |$\quad$|  |  |
| :--- | :--- |
| $v_{1}, v_{2}:$ | traffic flows in Directions 1 and 2 , in $\mathrm{pc} / \mathrm{h} ;$ |
| $l_{s}:$ | lost time for traffic release and vehicle startup, in s. |

## 4. DATA COLLECTION AND ANALYSIS

This step aimed to collect and analyze information related to the operational conditions of stop-and-go interventions along two-lane highways. Among the variables evaluated, the average travel speed of the vehicles in the work zone segment, the queue discharge and headway flow in sections upstream and downstream of the construction, the lost time between the passage of the last vehicle in the opposite direction and the startup of the first vehicle in the queue, in addition to local traffic characteristics such as classified vehicle count, were highlighted. The analyses provided a better understanding of the impacts of the works on the traffic flow operation and supported the subsequent steps of calibration of the traffic simulator and the proposed mathematical formulation.

In this study, data collection was performed in two ways. The first relied on the use of cameras positioned in the vicinity of the stop-and-go interventions to record the passing times of each vehicle at both ends of the intervention, as well as in upstream and downstream segments. The second considered the individual records of vehicles collected through a traffic analysis system (TAS) during the execution of works in the segment to identify the impacts of the work on the measures of effectiveness.

### 4.1. Data obtained in the field

The data collection in the field consisted of monitoring, for one day, a maintenance team of the MG-050 highway, under concession of AB Nascentes das Gerais. Cameras were installed at the ends of interventions, where the vehicle access into the segments is controlled during construction; additionally, the areas adjacent to the interventions were monitored. The traffic data collection took place on April 23, 2021, a Friday, from 08:00 h to 17:00 h, during which the maintenance team worked in three different locations between Formiga and Divinópolis:

- Work Zone 1: km 149, from 08:00 h to 10:30 h;
- Work Zone 2: km 177, from 12:20 h to 14:20 h;
- Work Zone 3: km 180, from 15:45 h to 17:30 h.

For each work zone, six cameras were installed, one at each access control point (two) and the others up to four km from the work segment in each direction, to evaluate the impacts of the downstream works on the formation of platoons upstream. As a reference, Points P3 and P4 designate the control points of the stop-and-go operation during the data collection.

Figure 5 shows the construction sites and the positioning of the cameras. It is important to note that the highway segments where the works are localized have heterogeneous characteristics, including different geometric conditions (upgrades, downgrades, and level terrain), such as the presence of speed bumps (Work Zone 2), tangents (Work Zone 1 and Work Zone 2) and horizontal curves (Work Zone 3).


Figure 5. Traffic data collection locations: Work 1 (a); Work 2 (b); and Work 3 (c)

Through filming, information was collected, such as traffic flow and composition, traffic data per stop-and-go cycle (vehicles served and duration), average vehicle speed on the segment in different situations (free, during construction, inside platoons and outside platoons),
percentage of vehicles in platoons and average headway of the different types of vehicles, to calculate the heavy vehicle equivalence factor. More details on the collection of traffic data are given in Andrade (2021).

### 4.2. TAS data

Data from a TAS were obtained from the concessionaire Arteris ViaPaulista during the execution of works on highway SP-255, near km 187, in the segment between Igaraçu do Tietê and Aparecida de São Manuel. This is a two-lane highway segment, in which pavement maintenance works were performed on March 30, 2020, with stop-and-go intervention, in Direction 1, between 07:40 h and 17:10 h .


Figure 6. Distributions of instantaneous speeds of vehicles measured in the TAS in direction 1, without (a) and with works (b); and in direction 2 , without (c) and with works (d).

The TAS individually records the passage of vehicles, detailing characteristics such as category, number of axles, weight, speed, and direction. To identify the period of operation of
the segment during stop-and-go operation, the intervals in which there are vehicles in only one direction, indicating the cycles of blocking and release of each lane, were verified. From these records, it was possible to perform comparative analyses of the speed for each situation (with and without construction) and vehicle category (pedestrian and trucks). Figure 6 shows the distribution curves of the instantaneous speed measured with the TAS for each direction in periods without and with works in the segment. In these graphs, the heavy vehicles were segregated into light ( 2 axels), medium ( 3 and 4 axels), heavy ( 5 and 6 axels), and extraheavy ( 7 and 9 axels).

Without interventions in the segment, distinct instantaneous speed curves were observed for each class of vehicles in Direction 1 due to the performance differences of the categories in upgrade sections. In the opposite direction, the classes of heavy vehicles have similar behaviors, with overlapping speed curves. Both graphs show that passenger cars travel at higher speeds. With ongoing work in the segment, the speeds are reduced according to the performance of the slowest vehicles (heavy and extraheavy trucks), with very reduced variations between the classes.

To obtain the desired speeds for each class of vehicles around construction sites, a parameter adopted in traffic microsimulation models, the values of the 85th percentile of the cumulative frequency distribution of the instantaneous speed in sections with construction work, rounded in multiples of 5 , were determined as follows:

- Car: $80 \mathrm{~km} / \mathrm{h}$;
- Light truck: $75 \mathrm{~km} / \mathrm{h}$;
- Medium truck: $70 \mathrm{~km} / \mathrm{h}$;
- Heavy truck: $70 \mathrm{~km} / \mathrm{h}$;
- Extraheavy truck: $65 \mathrm{~km} / \mathrm{h}$.


## 5. CALIBRATION AND VALIDATION OF AIMSUN NEXT

Microscopic Traffic simulators continuously analyze the individual conduct of each driver, adopting probabilistic distributions of drivers' behavioral characteristics and the different vehicle types with their peculiarities, such as power, weight, and acceleration (Barceló et al., 1995). The microscopic traffic simulator Aimsun Next, version 20, is based on three behavioral submodels, the car-following (Gipps, 1981), lane-changing (Gipps, 1986) and gap-acceptance (USDOT, 2005) submodels, and on a vehicle performance model, TWOPAS (Allen et al., 2000). The combination of these submodels allows the simulation of traffic streams under different traffic flow, geometry, and marking road situations.

The calibration of the traffic simulator was carried out to reproduce the conditions observed in the field, collected from the MG-050, in which three works were monitored at different locations and periods. For this, a network was built with the physical and operational characteristics of the existing road system, with emphasis on the geometric configuration of the deviation of works, such as the altimetry, number of lanes per segment, number of overtaking lanes, and vertical and horizontal alignments.

The traffic volumes observed in the data collection stage were entered into the simulator through origin-destination travel matrices for each 15-minute interval, segregated by vehicle type (cars and heavy vehicles) and by direction of travel (increasing and decreasing, in relation to the milestones of the highway), and the arrival of vehicles followed an exponential
distribution. To calibrate the performance model of heavy vehicles in Aimsun Next, heavy vehicles were subdivided into four categories: light, medium, heavy and extraheavy. For each category, cumulative mass and mass/power distribution curves were defined (Figure 7) based on data collected in the study by Lima et al. (2018), which include the weighing records of heavy vehicles obtained with portable truck scales installed along BR-040, in segments in Minas Gerais (between km 47 and km 767), and interviews conducted at the Federal Highway Police station (at km 554). In addition, the desired speeds of the vehicles in work zones were inserted based on the values obtained in the data collection stage.


Figure 7. Cumulative frequency distribution of Mass (a); and Mass/Power (b) of heavy vehicles. Adapted from Lima et al. (2018)

Once the vehicle characteristics and the desired speed distributions were calibrated, the next step consisted of simulating the scenarios of works 1 to 3 observed in the field, as previously mentioned, to achieve simulated measures of effectiveness like those collected in the field. Five replications were performed for each scenario. Table 1 shows the comparison of average travel speed data by type of vehicle (in motion) obtained through data collection and traffic microsimulation. The results show very similar values between the different approaches, which indicates that the simulation was able to reproduce the different traffic conditions observed well.

Table 1 - Average travel speed measurements comparison obtained in the field and from traffic microsimulation

| Work | Dir. | Passenger Car (km/h) |  |  | Heavy Vehicle (km/h) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Measured | Simulated | Difference | Measured | Simulated | Difference |
|  | 1 | 45.5 | 45.3 | 0\% | 38.1 | 37.5 | -2\% |
| Work 1 | 2 | 62.1 | 64.2 | 3\% | 58.4 | 58.9 | 1\% |
|  | 1 | 29.1 | 27.8 | -4\% | 26.6 | 26.8 | 1\% |
| Work 2 | 2 | 28.3 | 26.3 | -7\% | 24.6 | 24.7 | 1\% |
|  | 1 | 65.0 | 65.2 | 0\% | 58.1 | 62.2 | 7\% |
| Work 3 | 2 | 39.0 | 37.8 | -3\% | 34.0 | 35.2 | 3\% |

## 6. VALIDATION OF THE PROPOSED METHOD

The analysis of the results aimed to evaluate the behavior of the problem variables and to validate the results obtained through the proposed method. The use of simulation techniques to obtain experimental data has been increasingly frequent in the literature (Washburn et al., 2008; Jian et al., 2004) and allows the exploration of the different combinations of scenarios for the analysis of the results.

Thus, measures of effectiveness values for different configurations of demand and work zone lengths were obtained from the calibrated Aimsun Next software. A total of 28,350 simulations were simulated, resulting from the variation in parameters such as work zone length (from 500 to $5,000 \mathrm{~m}$ ), two-way traffic flow (from 200 to 1,000 vehicles per hour), percentage of heavy vehicles (from $20 \%$ to $50 \%$ ), percentage of the main directional traffic flow (from $50 \%$ to $70 \%$ ) and grades of the main direction ( $0 \%, 3 \%$, and $6 \%$ ), each combination replicated five times with different random seeds.

To reproduce the operation of a stop-and-go intervention in the simulator, traffic lights were inserted at each control point of the segment of work and activated by the presence of vehicles via detectors in the monitored road segments. Dynamically, the simulator keeps the traffic light green as long as there is a detection of vehicles in the respective approach. When no more vehicles are detected, the traffic light turns red, and the vehicles that have already entered the work zone segment complete the passage. After the last vehicle has passed, the traffic is released in the opposite direction, and the process is repeated.


Figure 8. Traffic network modeling in Aimsun Next software

Subsequently, some relevant variables for the application of the proposed method were determined, such as the heavy vehicle equivalence factor, saturation flow, and lost time. Table 2 indicates the reference values of these variables for level terrain. The values obtained for rolling and mountainous terrain, as well as the discussion of the methods for determining these variables, can be seen in Andrade (2021).

Table 2 - Values for level terrain

| Saturation flow | $Q_{1}=Q_{2}=1,850 \mathrm{pc} / \mathrm{h}$ |
| :--- | :--- |
| Total lost time | $I_{S}=8$ seconds |
|  | $P_{H V}=20 \%-E_{T}=2.64$ |
|  | $P_{H V}=25 \%-E_{T}=2.51$ |
|  | $P_{H V}=30 \%-E_{T}=2.40$ |
| Heavy vehicle equivalence factor | $P_{H V}=35 \%-E_{T}=2.31$ |
|  | $P_{H V}=40 \%-E_{T}=2.24$ |
|  | $P_{H V}=45 \%-E_{T}=2.19$ |
|  | $P_{H V}=50 \%-E_{T}=2.11$ |

Regarding the average travel speed in the work zone segment, there was a strong relationship between this variable and the work zone length and directional heavy vehicle flow.

It is possible to infer that the greater the extension of the work and/or the heavy vehicle flow is, the greater the likelihood of the formation of slow platoons and, consequently, the lower the average travel speeds in the segment. Table 3 indicates the value resulting from this analysis.

Table 3 - Average travel speed ( $\mathrm{km} / \mathrm{h}$ ) per heavy vehicle flow (veh/h) and work zone length ( m ) - Terrain: level ( $\mathrm{i}=0 \%$ )

| Heavy vehicle <br> flow (veh/h) | Work zone length (m) |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 25 | 58 | $\mathbf{1 , 0 0 0}$ | $\mathbf{1 , 5 0 0}$ | $\mathbf{2 , 0 0 0}$ | $\mathbf{2 , 5 0 0}$ | $\mathbf{3 , 0 0 0}$ | $\mathbf{3 , 5 0 0}$ | $\mathbf{4 , 0 0 0}$ | $\mathbf{4 , 5 0 0}$ | $\mathbf{5 , 0 0 0}$ |
| 50 | 56 | 58 | 59 | 59 | 59 | 59 | 58 | 58 | 58 | 58 |
| 75 | 55 | 57 | 57 | 58 | 58 | 58 | 57 | 57 | 57 | 57 |
| 100 | 54 | 56 | 56 | 56 | 55 | 55 | 55 | 55 | 54 | 54 |
| 125 | 54 | 55 | 55 | 55 | 55 | 55 | 54 | 54 | 54 | 54 |
| 150 | 53 | 55 | 55 | 55 | 54 | 54 | 54 | 53 | 53 | 53 |
| 175 | 53 | 55 | 55 | 54 | 54 | 54 | 53 | 53 | 53 | 53 |
| 200 | 53 | 54 | 54 | 54 | 54 | 53 | 53 | 53 | 52 | 52 |
| 225 | 53 | 54 | 54 | 54 | 54 | 53 | 53 | 53 | 52 | 52 |
| 250 | 53 | 54 | 54 | 54 | 53 | 53 | 52 | 52 | 52 | 52 |
| 275 | 53 | 54 | 53 | 53 | 53 | 52 | 52 | 52 | 51 | 51 |
| 300 | 52 | 53 | 53 | 53 | 52 | 52 | 52 | 51 | 51 | 51 |


(b)

(c)

Figure 9. Linear regression analysis of average cycle period (a); medium platoon size (b); and average delay (c), obtained through traffic simulation and the proposed method

The results of the simulations for each combination of work zone length, traffic flow, percentage of heavy vehicles, and percentage of the main directional traffic flow were compared with the values obtained from the application of the mathematical formulation proposed in this study. From the dispersion diagrams presented in Figure 9, linear regression techniques were applied to evaluate the following measures of effectiveness:

- Cycle time: interval between the passages of the first vehicle in the queue in the same direction in two consecutive periods.
- Average platoon size: ratio between the number of vehicles served in each direction and the number of cycles.
- Average delay time: difference between the average travel time of the approach to the work zone segment and the time in free flow conditions.
The comparison of the average cycle period results aims to verify whether the mathematical formulation generates results that are similar to the traffic simulation model. In this analysis, the coefficient of determination $\left(\mathrm{R}^{2}\right)$ was $99.9 \%$, with an average deviation of only $1.3 \%$. Similarly, the evaluations of the measures of effectiveness also showed satisfactory results. The coefficients of determination $\mathrm{R}^{2}$ were greater than $99 \%$ in both cases. The average deviations were only $1.8 \%$ for the platoon size and $0.4 \%$ for the average delay time.


## 7. FINAL REMARKS

The main goal of this study was to propose a method for determining capacity and measures of effectiveness of two-lane highways segments under stop-and-go operation during the execution of construction works. For this purpose, a deterministic mathematical formulation was developed based on queuing theory and similarities with the dissipation of vehicles via traffic lights.

The developed method allows the average delays of the users and the average sizes of the platoons formed in each stop-and-go cycle to be accurately estimated, as well as details the different stages involved in the process, such as the effective green and clearance intervals. Through the formulation presented, it was also possible to calculate the segment capacity and dimension of the maximum length of the work zone, conditioned to acceptable platoon size and average delay limits.

To validate the method, a traffic microsimulation network was built in the Aimsun Next software, which allowed obtaining measures of effectiveness for several hypothetical scenarios. The comparison of the results of the two approaches showed very similar values.

For future studies, it is suggested to extend the stage of validation of the results to also evaluate the formulations described for calculating the capacity and maximum length of work zones. In addition, it is recommended to identify parameters and service measures to determine the level of service of two-lane highway segments during the execution of construction works to classify the different traffic conditions and establish minimum criteria for the operation of the intervention. From this, it will be possible to limit the execution times and the length of the work zone segment, according to the traffic conditions in the segment.

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